Q #14

$y = \tan(2x)$

$y'\left(\frac{\pi}{6}\right)$

$y'(x) = \sec^2(2x) \cdot 2$

$= 2\sec^2(2x)$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(2x) = 2$

$y'\left(\frac{\pi}{6}\right) = \frac{2}{\cos^2\left(2\cdot\frac{\pi}{6}\right)} = \frac{2}{\left[\cos\left(\frac{\pi}{3}\right)\right]^2} = \frac{2}{\left(\frac{1}{2}\right)^2} = 8$

6a. there exists a number $h \in [0,1]$ such that $g(h) \geq g(x)$ for all $x \in [0,1]$

$\Rightarrow$ there is a maximum

(3+1) If the line $3x-4y=0$ is tangent in the first quadrant to $y=x^3+k$, then $k = \ldots\ldots?$

(1) slopes of tangent lines are "connected" with values of derivatives...

slope of the tangent line:

$$3x-4y=0$$
$$3x=4y$$
$$\frac{3}{4}x=y$$
$$\therefore m=\frac{3}{4}$$

(2) $f(x)=x^3+k$

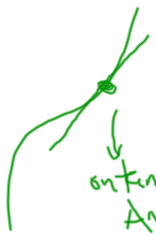
$$\Rightarrow f'(x)=3x^2$$

set deriv $=\frac{3}{4}$ & solve for x:

$$3x^2=\frac{3}{4}$$
$$x^2=\frac{1}{4}$$
$$x=\pm\frac{1}{2}$$

(3) Q1 $\Rightarrow$ $x=\frac{1}{2}$

$$y=\frac{3}{4}(x)=\frac{3}{4}\left(\frac{1}{2}\right)=\frac{3}{8}=(x^3+k)=\left(\frac{1}{2}\right)^3+k$$
$$\frac{3}{8}=\frac{1}{8}+k$$
$$\frac{3}{8}-\frac{1}{8}=\frac{2}{8}=k$$
$$\frac{1}{4}$$



Q11) $f(x) = 2 - 4x - 3x^2$

$$\lim_{w \to x} \frac{f(w) - f(x)}{w - x} \qquad \frac{(-3w^2 - 4w + 2) - (-3x^2 - 4x + 2)}{w - x}$$

$$\frac{-3w^2 - 4w + 3x^2 + 4x}{w - x} \qquad \frac{(-3w^2 + 3x^2) + (-4w + 4x)}{w - x}$$

$$\frac{-3(w^2 - x^2) - 4(w - x)}{w - x} \qquad \frac{-3(w^2 - x^2)}{w - x} - \frac{4\cancel{(w - x)}}{\cancel{w - x}}$$

$$\frac{-3\cancel{(w - x)}(w + x)}{\cancel{w - x}} - 4 \qquad -3(w + x) - 4$$

so $\lim_{w \to x} -3(w+x) - 4 = -3(x + x) - 4$

$$-3(2x) - 4 \quad \boxed{-6x - 4}$$

Q9

$$\lim_{x\to 1} \frac{\sqrt{x}-x}{x-1} = \lim_{x\to 1} \left(\frac{\sqrt{x}-x}{x-1}\right)\left(\frac{\sqrt{x}+x}{\sqrt{x}+x}\right)$$

conjugate

$$= \lim_{x\to 1} \frac{\overset{x(1-x)}{x-x^2}}{(x-1)(\sqrt{x}+x)} \ldots$$

8 | Find derivative of $f(x) = x\left(\sqrt{x}+x\right)^4$

$$f'(x) = (1)\left(\sqrt{x}+x\right)^4 + (x)\frac{d}{dx}\left(\left(\sqrt{x}+x\right)^4\right)$$

$$= \left(\sqrt{x}+x\right)^4 + x\left[4\left(\sqrt{x}+x\right)^3 \frac{d}{dx}\left(\sqrt{x}+x\right)\right]$$

$x^{1/2}$

$$= \left(\sqrt{x}+x\right)^4 + 4x\left(\sqrt{x}+x\right)^3\left[\frac{1}{2}x^{-\frac{1}{2}} + 1\right]$$